8.7 Coherence (scalar theory)

Mutual coherence function	$\Gamma_{12}(\tau)=\langle\psi_1(t)\psi_2^*(t+\tau)\rangle$	(8.97)	Γ_{ij} mutual coherence function τ temporal interval ψ_i (complex) wave disturbance at spatial point i
Complex degree of coherence	$\gamma_{12}(\tau)=\dfrac{\langle\psi_1(t)\psi_2^*(t+\tau)\rangle}{[\langle\|\psi_1\|^2\rangle\langle\|\psi_2\|^2\rangle]^{1/2}}$	(8.98)	t time $\langle\cdot\rangle$ mean over time γ_{ij} complex degree of coherence * complex conjugate
	$=\dfrac{\Gamma_{12}(\tau)}{[\Gamma_{11}(0)\Gamma_{22}(0)]^{1/2}}$	(8.99)	
Combined intensity[a]	$I_{\mathrm{tot}}=I_1+I_2+2(I_1I_2)^{1/2}\Re[\gamma_{12}(\tau)]$	(8.100)	I_{tot} combined intensity I_i intensity of disturbance at point i $\Re$ real part of
Fringe visibility	$V(\tau)=\dfrac{2(I_1I_2)^{1/2}}{I_1+I_2}\|\gamma_{12}(\tau)\|$	(8.101)	
if $\|\gamma_{12}(\tau)\|$ is a constant:	$V=\dfrac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}$	(8.102)	$I_{\max}$ max. combined intensity $I_{\min}$ min. combined intensity
if $I_1=I_2$:	$V(\tau)=\|\gamma_{12}(\tau)\|$	(8.103)	
Complex degree of temporal coherence[b]	$\gamma(\tau)=\dfrac{\langle\psi_1(t)\psi_1^*(t+\tau)\rangle}{\langle\|\psi_1(t)^2\|\rangle}$	(8.104)	$\gamma(\tau)$ degree of temporal coherence $I(\omega)$ specific intensity ω radiation angular frequency c speed of light
	$=\dfrac{\int I(\omega)\mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\omega}{\int I(\omega)\,\mathrm{d}\omega}$	(8.105)	
Coherence time and length	$\Delta\tau_{\mathrm{c}}=\dfrac{\Delta l_{\mathrm{c}}}{c}\sim\dfrac{1}{\Delta\nu}$	(8.106)	$\Delta\tau_{\mathrm{c}}$ coherence time Δl_{c} coherence length $\Delta\nu$ spectral bandwidth
Complex degree of spatial coherence[c]	$\gamma(\boldsymbol{D})=\dfrac{\langle\psi_1\psi_2^*\rangle}{[\langle\|\psi_1\|^2\rangle\langle\|\psi_2\|^2\rangle]^{1/2}}$	(8.107)	$\gamma(\boldsymbol{D})$ degree of spatial coherence $\boldsymbol{D}$ spatial separation of points 1 and 2 $I(\hat{\boldsymbol{s}})$ specific intensity of distant extended source in direction $\hat{\boldsymbol{s}}$ $\mathrm{d}\Omega$ differential solid angle $\hat{\boldsymbol{s}}$ unit vector in the direction of $\mathrm{d}\Omega$
	$=\dfrac{\int I(\hat{\boldsymbol{s}})\mathrm{e}^{\mathrm{i}k\boldsymbol{D}\cdot\hat{\boldsymbol{s}}}\,\mathrm{d}\Omega}{\int I(\hat{\boldsymbol{s}})\,\mathrm{d}\Omega}$	(8.108)	
Intensity correlation[d]	$\dfrac{\langle I_1I_2\rangle}{[\langle I_1\rangle^2\langle I_2\rangle^2]^{1/2}}=1+\gamma^2(\boldsymbol{D})$	(8.109)	k wavenumber
Speckle intensity distribution[e]	$\mathrm{pr}(I)=\dfrac{1}{\langle I\rangle}\mathrm{e}^{-I/\langle I\rangle}$	(8.110)	pr probability density
Speckle size (coherence width)	$\Delta w_{\mathrm{c}}\simeq\dfrac{\lambda}{\alpha}$	(8.111)	Δw_{c} characteristic speckle size λ wavelength α source angular size as seen from the screen

[a]From interfering the disturbances at points 1 and 2 with a relative delay τ.
[b]Or "autocorrelation function."
[c]Between two points on a wavefront, separated by $\boldsymbol{D}$. The integral is over the entire extended source.
[d]For wave disturbances that have a Gaussian probability distribution in amplitude. This is "Gaussian light" such as from a thermal source.
[e]Also for Gaussian light.